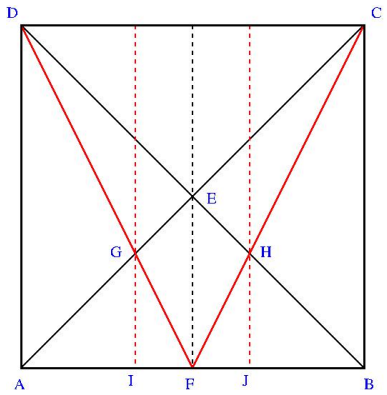
Extra proof problem for Chapter 6

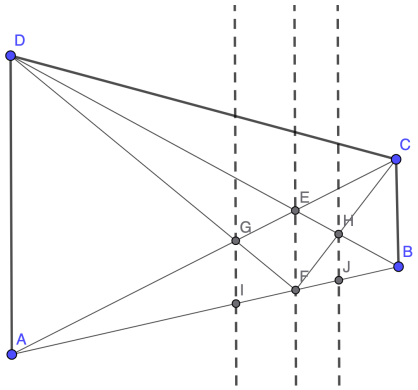
This exercise asks you to either write a justification for why the trisection of the door in Exercise 6.28 can be done the way one group outlined it, or else find a reason why this construction doesn't work. We have made your job easier by splitting up the problem into two pieces, both of which need to be valid if the argument is to work.

(a) Does their construction work on a perfect square? The diagram is copied in below. Given a square $ABCD$, the diagram shows diagonals AC and BD meeting at E, a dashed vertical line through E meeting AB at F. The point G is the intersection of AE and DF, the point H is the intersection of BE and FC, and I and J are the intersection of AB with vertical lines (dashed) through G and H respectively.



Prove or refute that AI, IJ and JB are equal in this case. Your methods can include what you remember from analytic geometry (coordinate geometry) in the plane.

(b) For the general case, a similar construction is shown below. We suppose that now $ABCD$ is a perspective image of a square. Use what you know about mesh maps to argue why AI, IJ and JB should or shouldn't be equal.



(c) Is there anything you had to assume, and if so, was it true given the assumptions of Exercise 6.28?